Equilibrium Evolution in a Two-Echelon Supply Chain with Financially Constrained Retailers: The Impact of Equity Financing

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Abstract: This paper considers a two-echelon supply chain that has a supplier and two capital constrained retailers and in which the retailers compete in a Cournot fashion. We study the impact of external financing on the players' optimal decisions and supply chain performance. We show that as competition intensity increases, the supplier (as the Stackelberg leader) may consider merging with one retailer to avoid double marginalization. Yet, the deselected retailer may utilize external financing to return to the supply chain. We explicitly model the evolution of equilibrium scenarios and identify the conditions under which the supplier may prefer to provide trade credit to only one retailer and the other retailer may use external financing. We also carry out extensive sensitivity analyses with respect to a retailer's capital structure and the retailer's competition intensity.

Keywords: trade credit; equity financing; Cournot competition; capital constraints.

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1. Introduction

One of the most important decisions that capital constrained retailers face is how to finance their operations. In response to the retailers' financial constraints, many manufacturers extend trade credit to their buyers so that retail channels can maintain sustainable operations. Through this credit, buyers can delay their payments until the products are sold. Trade credit has become one of the most popular financing mechanisms in today's business practices. For example, over 80% of B2B transactions in the UK are made on trade credit. Furthermore, in the US, approximately 80% of firms offer their products on trade credit (Seifert et al., 2013). A supplier endowed with sufficient capital provides trade credit to help retailers stay in business and achieve a win-win situation (Goyal, 1985; Huang, 2007; Teng et al., 2012; Taleizadeh et al.,2013). Indeed, many scholars have shown that all channel members may benefit from the use of trade credit, as opposed to the use of bank loans, to finance capital constrained retailers (Jing et al., 2012; Jing and Seidmann, 2014; Yang et al., 2014; Du et al., 2013; Kouvelis and Zhao, 2012).

Research has shown that in a one-to-many supply chain with a supplier and multiple retailers, each player becomes worse off when the competition between retailers is intensified (Yang and Zhou, 2006; Wu et al., 2012). As a consequence, manufacturers may strategically streamline their retail channels by cooperating with a single retailer. This implies that other retailers may drop out of business. Glock and Kim (2015) study such a supply chain and confirm that some retailers may be deselected by the vendor. Yang and Zhou (2006) study the effect of retailers' competitive paradigms on each player's equilibrium behavior. They find that if two retailers collude, all of the supply chain members make more profits. The collusion between retailers is essentially equivalent to the case in which a single retailer exists. Thus, the above result implies that manufacturer can benefit from merging with a single retailer. In addition, numerous studies concerning competition between supplier and retailer interaction in a supply chain have been conducted. Li et al. (2015) shed light on issues regarding conflict between two heterogeneous channels and vertical competition. Wang et al. (2016) establish a linear demand model to explore the

channel selection and pricing strategy in a supply chain comprising a dominant multi-channel retailer and a manufacturer. In addition, the retailer is the leader of pricing in the supply chain.

Notably, the above findings build on an important assumption: each player has sufficient capital to support its various business decisions. As discussed earlier, however, many firms suffer from capital shortage. In this paper, we are particularly interested in the situation in which retailers have capital constraints and may receive trade credit from their supplier. For example, through our interaction with Telstra, the largest communication service provider in Australia, we find that many retail outlets that sell their mobile devices and service plans are small buyers and have very limited financial capacity. Because the sales of these small retail outlets account for a large portion of total sales, Telstra must provide these retailers with trade credit.

Some literature has established that in a one-to-one supply chain, the coordination between the supplier and the capital constrained retailer can overcome the double marginalization effect and improve the profits of the overall supply chain and individual firms (Spengler, 1950; Cachon, 2003; Ru and Wang, 2010). Lee and Rhee (2011) show that full coordination can be achieved when a supplier properly designs the trade credit and buyback contract. Chen (2015) shows that a capital constrained retailer makes more profit in a centralized supply chain than in a decentralized supply chain. Feng et al. (2015) investigate the supply chain coordination problem under three contract forms, including revenue sharing, buy back and revenue-sharing-and-buy-back contracts, when both the supplier and the retailer are subject to capital constraint. They show that the revenue sharing and buy back contracts are ineffective under certain budget scenarios, whereas the revenue-sharing-and-buy-back contract can always coordinate the supply chain and arbitrarily divide the total profit among players (Lee and Rhee, 2010; Yan and Sun, 2013; Xu et al., 2015; Zhang et al., 2014; Jin et al., 2015).

Although many papers have documented the benefit of the merger of a supplier and a single (financially constrained) retailer, other retailers may still have opportunities to join the supply chain. In practice, other external financing options,

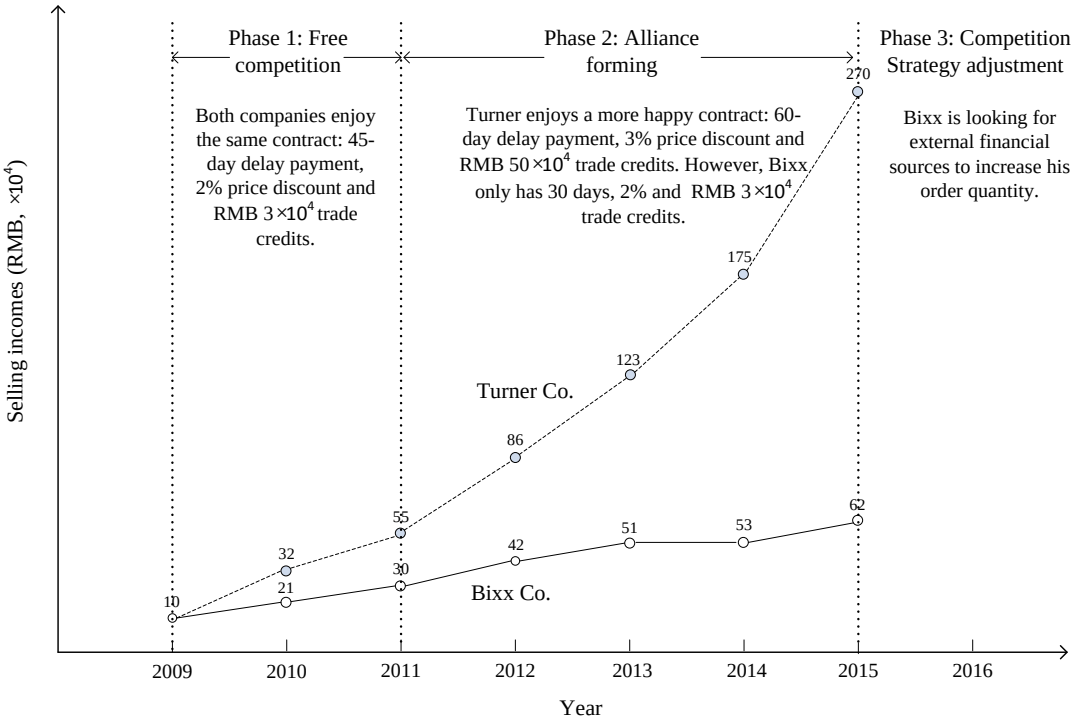
such as equity financing, are available to these retailers. The retailer that drops out of the supply chain can use equity financing to resolve capital constraints so that it can still return to the supply chain. Brander and Lewis (1986) use a duopoly model to study the impact of firms' capital structure on ordering decisions. They indicate that firms with greater debt tend to be more aggressive, an effect termed as "the limited liability effect". In equilibrium, such firms have strategic advantages in the competitive market. To extend these findings, Brander and Lewis (1988) incorporate bankruptcy costs into the competition model and focus on the effect of debt levels on firms' equilibrium behaviors.

In this paper, we consider a two-echelon supply chain that has a supplier and two capital constrained retailers and in which the retailers engage in quantity competition. The supplier, as a leader in the Stackelberg game, can provide trade credit to the capital constrained retailers. Several papers employing a similar setup without capital constraints have shown that as competition intensity increases, each player's profit decreases (Yang and Zhou, 2006; Fang and Shou, 2015). As the Stackelberg leader, the supplier may find it beneficial to merge with one retailer to avoid double marginalization. In this case, the supplier provides trade credit to only one retailer, and the other retailer is not financed. If no external financing source is available, this retailer will eventually drop out of the supply chain. However, as mentioned earlier, the retailer may have access to equity financing or bank financing (i.e., debt financing). This poses an interesting question: In the presence of external financing, can the abandoned retailer rejoin the supply chain? In other worlds, what is the impact of external financing on supply chain performance as a whole and on each player's profit? Several papers have studied the effect of bank financing on operational performance in a one-to-one supply chain. When a financially constrained retailer uses bank loans, the supply chain's efficiency cannot be improved; thus, coordination cannot be fully achieved (Kouvelis and Zhao, 2012; Chen, 2015).

Our paper is distinct in that it considers a competitive setting in which downstream retailers are subject to financial constraints. The supplier provides trade credit to one or two retailers, and the retailer endowed with no support from the

supplier may seek finance from investors and/or banks. With this setup, we consider three cases: in Case 1, the supplier provides trade credit to both retailers; in Case 2, the supplier provides trade credit to only one retailer and the other retailer has no other financing options; and in Case 3, the supplier provides trade credit to one retailer but the other retailer is able to use external financing, including equity financing and debt financing.

Through our discussions with an SME firm in China, we observe the evolution of a pattern of competition similar to that in the above three cases. Turner Co. and Bixx Co. both sell art material products in China. They began to sell Canson branded products supplied by Arjowiggins Co. in 2009. Figure 1 depicts the revenues of the two companies from 2009 to 2015. The figure shows scenarios that are similar to those considered in this paper. During 2009-2011, both companies were offered the same contract involving a 45-day delay payment, 2% price discount and RMB 30,000 trade credits. The two companies enjoyed a similar profit rate of approximately 25% during that period. This phase of competition resembles Case 1, in which retailers engage in free competition. Thanks to proactive market exploration, after 2011, Turner Co. had significantly increased its sales. To compensate Turner Co., the supplier started to offer the retailer a better contract with a 60-day delay payment, 3% discount and RMB 500,000 trade credits. By contrast, Bixx Co. was offered an even worse contract than that it previously had, with only a 30-day delay payment. As a result, in 2015, the profit of Turner Co. grew to RMB 2,700,000, which is equivalent to the profit rate of 29%. By contrast, Bixx Co. maintained nearly the same profit rate that it previously had. In this phase, the supplier had formed a strategic alliance with Turner Co., which resembles Case 2 in our model. Currently, Bixx Co. is proactively searching for external capital from banks and investors with the hope of catching up with Turner Co.



Data source: Turner Co.'s market analysis report.

Fig. 1. Revenue growth of Turner Co. and Bixx Co. from 2009 to 2015 (data obtained from a market analysis report provided by the CEO of Turner Co.)

After setting up the model, we start by characterizing the equilibrium for each case and then make a complete comparison among the three cases to examine the impact of external financing. We also study how the second retailer's capital structure and the competition intensity affect equilibrium evolution and competitive behaviors.

The contributions of this paper are summarized as follows. First, to our knowledge, this paper is among the first to study trade credit and external financing in a supply chain with competing capital constrained retailers. We contribute by analyzing the influence of competition intensity on the players' optimal decisions. Second, many papers have studied the trade-off between trade credit and bank financing in the supply chain literature. The current paper departs from this literature by incorporating equity financing into a competition model, enabling us to study how a retailer's capital structure affects supply chain performance. Interestingly, we show that an optimal ratio of equity financing exists, in which the supplier may be better off when the retailer drops out of business. Third, by comparing the equilibria of the three cases, we fully characterize the evolution of equilibrium dynamics.

The remainder of the paper is organized as follows. The next section sets up the model. Section 3 analyzes the equilibria of the three cases. Section 4 examines the impacts of the equity financing ratio and competition intensity on the players' optimal decisions. We also numerically verify the results derived in this paper. Finally, Section 5 concludes.

2. Model Setup

We consider a two-echelon supply chain with a supplier (denoted by S) and two symmetric retailers (denoted by R_1 and R_2). The retailers have no capital endowment and must rely on some capital sources to finance their operations. As the Stackelberg leader, the supplier determines wholesale price. As Stackelberg followers, the two retailers determine their order quantities given the wholesale price charged by the supplier. Then, the retailers sell the products in a consumer market. At the end of the period, all the financial transactions will be made. We consider the Cournot competition, in which the inverse demand function of retailer R_i can be written as follows: $p_i^k = A - q_i^k - \gamma q_j^k$, where p_i^k denotes the price of retailer R_i's product in Case k, and q_i^k and q_j^k denote retailer R_i's and R_j's order quantity in Case k, respectively. The parameter $\gamma \in [0,1]$ represents the degree of substitution between the competing products. A larger value of γ implies more intense competition. In particular, the case of $\gamma = 0$ represents independent products and the case of $\gamma = 1$ represents perfect substitutes (Aray, 2008; Aray and Mittendorf, 2013; Cho, 2014; Glock and Kim, 2015).

The supplier provides trade credit to one or two retailers. If the supplier awards trade credit to only one retailer, e.g., retailer R_1, retailer R_2 may be able to seek external financing. In summary, we consider the following three cases:

Case 1: The supplier provides trade credit to two retailers, which place orders with the supplier on trade credit. The supplier sets the wholesale price w_1 per unit with fixed cost c. The retailers determine their order quantities q_1^1 and q_2^1 and sell the products to their customers at prices p_1^1 and p_2^1.

Case 2: Note that the two suppliers are *ex ante* symmetric. Without loss of generality, assume that the supplier provides trade credit to only retailer R_1. Retailer R_2 does not have the privilege to use external financing. Thus, it simply drops out of the supply chain. In this case, we study the game between the supplier and retailer R_1, in which the revenue sharing contract is used to coordinate the channel (Cachon, 2003; Cachon and Lariviere, 2005).

Case 3: Unlike in Case 2, in this case, the retailer is able to seek financial support from both investors and banks. In other words, retailer R_2 uses external financing sources by transferring a certain proportion of its shares to investors and borrowing loans from a bank. Assume that the equity financing ratio is ϕ and that the remaining portion $1-\phi$ is the bank debt, where $\phi \in (0,1)$.

Table 1 shows capital sources and competition structures for the above three cases. Furthermore, we summarize the models considered in this paper in Fig. 2.

Table 1 The capital sources and competition structures

Case	R_1	R_2	Competition structure
Case 1	*TC* from *S*	*TC* from *S*	S (leader), R_1 and R_2 (followers) Cournot competition
Case 2	*TC* from *S*	drop out	S (leader), R_1 (follower)
Case 3	*TC* from *S*	equity and bank financing	S (leader), R_1 and R_2 (followers) Cournot competition

Note: We use "*TC*" to represent trade credit.

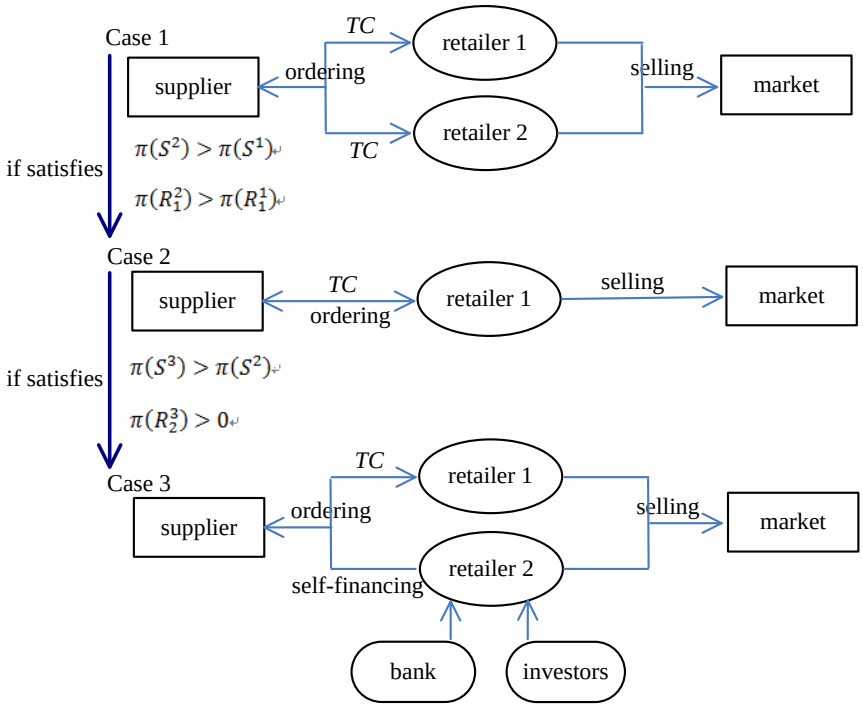


Fig. 2. The evolution process of the three equilibrium scenarios

To ease interpretation, we list the parameters and the decision variables below.

Parameters:

c Unit production cost.

p_i^k Sale price charged by retailer R_i in Case k, where i=1,2 and k=1,2,3.

r_s Interest rate of trade credit.

r_f Interest rate of bank loans.

γ Competition intensity.

α Revenue sharing rate given to the retailer in Case 2.

ϕ Equity financing ratio in Case 3.

$\pi(R_i^k)$ Retailer R_i's profit in Case k.

$\pi(S^k)$ The supplier's profit in Case k.

$\pi(T_k)$ The supply chain's profit in Case k.

Decision Variables:

q_i^k Retailer R_i's order quantity in Case k.

w_k Wholesale price in Case k.

3. Equilibrium Analysis

In this section, we examine the retailers' ordering decisions and the supplier's pricing decision for each case. As discussed earlier, the supplier in our models is the Stackelberg leader, while the retailers are the Stackelberg followers. By using the backward induction approach, we first look at the retailers' problems and then examine the supplier's decision regarding wholesale prices.

3.1 Case 1: The supplier provides trade credit to both retailers

In this case, the supplier provides trade credit to both retailers. We denote the supplier's wholesale price as w_1 and the retailers' order quantities as q_1^1 and q_2^1. Retailer R_i determines its order quantity q_i^1 and is required to pay $w_1 q_i^1(1+r_s)$ to the supplier at the end of the selling period. The sale price is denoted by p_i^1. Thus, retailer R_i's profit in Case 1 is

$$\pi(R_i^1) = p_i^1 q_i^1 - w_1 q_i^1(1+r_s) \tag{1}$$

The supplier receives payment of $w_1(q_1^1+q_2^1)(1+r_s)$ from two retailers. At the same time, the supplier's production cost for $q_1^1+q_2^1$ units of products is $c(q_1^1+q_2^1)(1+r_f)$, including the risk-free interest charged by financial institutions. Thus, the supplier's profit in Case 1 is

$$\pi(S^1) = w_1(q_1^1+q_2^1)(1+r_s) - c(q_1^1+q_2^1)(1+r_f) \tag{2}$$

Following the standard backward induction approach, we start by characterizing the equilibrium of the retailers. Then, taking account of the retailers' best responses, the supplier maximizes its profit by choosing the optimal wholesale price. The following results characterize the equilibrium decisions of the supplier and the two retailers.

Lemma 1. *In the case in which the supplier provides trade credit to both retailers, the equilibrium wholesale price and order quantities are* $w_1^* = \frac{1}{2(1+r_s)}\big(A + c(1+r_f)\big)$ *and* $q_1^{1*} = q_2^{1*} = \frac{1}{2(2+\gamma)}\big(A - c(1+r_f)\big)$, *respectively.*

All proofs are provided in the Appendix. Due to symmetry, two retailers have equal order quantities and the sale prices are the same, i.e., $p_1^{1*} = p_2^{1*} = \frac{1}{2(2+\gamma)}\big((3+\gamma)A - (1+\gamma)c(1+r_f)\big)$. Therefore, they have the same profit in equilibrium:

$\pi(R_1^1) = \pi(R_2^1) = \frac{1}{4(2+\gamma)^2}\left(A - c(1+r_f)\right)^2$. The supplier's profit is $\pi(S^1) = \frac{1}{2(2+\gamma)}\left(A - c(1+r_f)\right)^2$. In this case, from the supplier's perspective, two retailers are equivalent to an aggregate retailer. Thus, the wholesale price is independent of competition intensity. By contrast, the order quantities decrease as competitive intensity increases. As shown, the profits of the supplier, the retailers, and the supply chain all decrease with increased competition intensity.

3.2 Case 2: The supplier merges with retailer R_1

In this case, the supplier does not provide trade credit to retailer R_2, and retailer R_2 does not have access to external financing, and, therefore, will not participate in the supply chain. Retailer R_1's sale price is a linear function of demand, i.e., $p_1^2 = A - q_1^2$. We first consider a wholesale price contract. Similar to Case 1, we obtain the wholesale price $w_2 = \frac{1}{2(1+r_s)}\left(A + c(1+r_f)\right)$ and retailer R_1 's order quantity $q_1^2 = \frac{1}{4}\left(A - c(1+r_f)\right)$. The supplier's profit is $\pi(S^2) = \frac{1}{8}\left(A - c\left(1+r_f\right)\right)^2$, and retailer R_1's is $\pi(R_1^2) = \frac{1}{16}\left(A - c\left(1+r_f\right)\right)^2$. One can easily see that $\pi(S^1) > \pi(S^2)$, $\pi(R_1^1) \leq \pi(R_1^2)$ and $\pi(S^1) + \pi(R_1^1) > \pi(S^2) + \pi(R_1^2)$. Hence, the supplier prefers not to merge with the retailer under the wholesale price contract. Next, we consider a revenue sharing contract between the supplier and retailer R_1. If the supplier is better off in Case 2, then a merger may be achieved.

First consider the scenario in which the supply chain is centralized, namely, the supplier and retailer R_1 are fully integrated. The total supply chain profit with a centralized decision is given by $\pi(T_2) = p_1^2 q_1^2 - c q_1^2(1+r_f)$. Retailer R_1's optimal order quantity in the case of a centralized decision is $q_1^{2*} = \frac{1}{2}\left(A - c(1+r_f)\right)$. Then, we study the case in which the supply chain is decentralized. Assume that the retailer's revenue sharing ratio is given by $\alpha \in [0,1]$ and that the wholesale price is w_2. Specifically, the retailer's share of sales revenue is α, and the remaining $1-\alpha$ goes to the supplier. Retailer R_1 determines its order quantity, which is equal to the optimal order quantity under the centralized decision. Retailer R_1's and the supplier's

profits in Case 2 are

$$\pi(R_1^2) = \alpha p_1^2 q_1^2 - w_2 q_1^2 (1 + r_s) \tag{3}$$

$$\pi(S^2) = (1-\alpha) p_1^2 q_1^2 + w_2 q_1^2 (1 + r_s) - c q_1^2 (1 + r_f) \tag{4}$$

Solving this problem, we obtain the optimal decisions of the supplier and retailer R_1 in Lemma 2.

Lemma 2. *In the case in which retailer R_2 drops out of the supply chain, the optimal wholesale price is $w_2^* = \frac{\alpha c(1+r_f)}{(1+r_s)}$ and the optimal ordering quantity is $q_1^{2*} = \frac{1}{2}\left(A - c(1+r_f)\right)$.*

Base on Lemma 2, we can further show that retailer R_1's and the supplier's profits are $\pi(R_1^2) = \frac{\alpha}{4}\left(A - c\left(1+r_f\right)\right)^2$ and $\pi(S^2) = \frac{1-\alpha}{4}\left(A - c\left(1+r_f\right)\right)^2$, respectively. Obviously, the order quantity of retailer R_1 is greater than that in Case 1, i.e., $q_1^{2*} > q_1^{1*} + q_2^{1*}$. The appropriate revenue sharing ratio needs to be used to make both the supplier and retailer R_1 better off than in Case 1.

3.3 Case 3: Retailer R_2 enters the market with external financing

In this case, the supplier still provides trade credit to only retailer R_1, but retailer R_2 uses external financing to support its operations. Thus, retailer R_1 and retailer R_2 engage in Cournot competition. Consistent with our observations, the supplier offers the same wholesale price to both retailers. Retailer R_1 determines its order quantity q_1^3 and is required to pay $w_3 q_1^3 (1 + r_s)$ to the supplier at the end of the selling period. Retailer R_1's profit in Case 3 is

$$\pi(R_1^3) = p_1^3 q_1^3 - w_3 q_1^3 (1 + r_s) \tag{5}$$

For retailer R_2's external financing, we assume that the equity financing ratio is ϕ, while the remaining ratio $1 - \phi$ is the bank debt. Given the order quantity q_2^3, retailer R_2 should finance the amount $\phi w_3 q_2^3$ from investors and borrow $(1-\phi) w_3 q_2^3$ from a bank. At the end of the selling period, retailer R_2 transfers a fraction ϕ of realized sales revenue as a return to investors. Thus, retailer R_2's profit in Case 3 is

$$\pi(R_2^3) = (1-\phi)(p_2^3 q_2^3 - (1-\phi) w_3 q_2^3 (1 + r_f)) \tag{6}$$

The supplier's production cost for $q_1^3 + q_2^3$ units of products is $c(q_1^3 + q_2^3)\left(1 + r_f\right)$,

including the risk-free interest charged by financial institutions. The supplier receives a payment of $w_3 q_1^3(1+r_s)$ from retailer R_1 at the end of the selling period. The supplier receives a payment of $w_3 q_2^3$ from retailer R_2 at the beginning of the sale period, which is equivalent to $w_3 q_2^3(1+r_f)$ at the end of the selling period when time value is considered. Therefore, the supplier's profit in Case 3 is

$$\pi(S^3) = w_3 q_1^3(1+r_s) + w_3 q_2^3\left(1+r_f\right) - c(q_1^3+q_2^3)(1+r_f) \tag{7}$$

Following a similar logic as that employed in Case 1, we first examine the retailers' ordering behaviors and then study the supplier's optimal wholesale price. We provide the optimal decisions of the players in Lemma 3.

Lemma 3. *In the case in which the supplier provides trade credit to only retailer R_1 and retailer R_2 uses external financing, the supplier sets the optimal wholesale price as* $w_3^* = \frac{c(B+D)\left(1+r_f\right)-\left(2+r_f+r_s\right)(2-\gamma)A}{2\left(B(1+r_s)+D\left(1+r_f\right)\right)}$. *Given this wholesale price, the retailers' optimal order quantities are given by* $q_1^{3*} = \frac{((2-\gamma)A+Bw_3)}{4-\gamma^2}$ *and* $q_2^{3*} = \frac{((2-\gamma)A+Dw_3)}{4-\gamma^2}$, *respectively, where* $B = \gamma(1-\phi)\left(1+r_f\right) - 2(1+r_s)$ *and* $D = \gamma(1+r_s) - 2(1-\phi)\left(1+r_f\right)$.

Based on Lemma 3, we can show that retailer R_1's and retailer R_2's profits in Case 3 are $\pi(R_1^3) = \left(\frac{[B(r_s-r_f)+2D\left(1+r_f\right)]A-B\left((1-\phi)\left(1+r_f\right)+(1+r_s)\right)c\left(1+r_f\right)}{\left[2(2+\gamma)\left(B(1+r_s)+D\left(1+r_f\right)\right)\right]}\right)^2$ and $\pi(R_2^3) = (1-\phi)\left(\frac{[D(r_f-r_s)+2B(1+r_s)]A-D\left((1-\phi)\left(1+r_f\right)+(1+r_s)\right)c\left(1+r_f\right)}{\left[2(2+\gamma)\left(B(1+r_s)+D\left(1+r_f\right)\right)\right]}\right)^2$. The supplier's profit is given by $\pi(S^3) = \frac{\begin{array}{c}2(2-\gamma)\left(B\left(\left(1+r_f\right)+5(1+r_s)\right)+D\left(5\left(1+r_f\right)+(1+r_s)\right)\right)Ac\left(1+r_f\right)^2 \\ +(2-\gamma)(2+\mathrm{r_f}+\mathrm{r_s})^2A^2+(B+D)^2\left(c\left(1+r_f\right)\right)^2\end{array}}{-4(4-\gamma^2)\left(B(1+r_s)+D\left(1+r_f\right)\right)}$. In the next section, we further analyze the impact of the equity financing ratio and competition intensity on the optimal decisions and each player's profit.

4. Impacts of the Equity Financing Ratio and Competition Intensity

In this section, we first investigate how the equity financing ratio affects the

equilibrium wholesale price and order quantities. Next, we compare the supplier's wholesale price and the retailers' order quantities across all cases. We then focus on the effect of the equity financing ratio on each player's profit. Finally, we analyze the effect of competitive intensity on the equilibrium result and equilibrium evolution.

4.1 The Equity Financing Ratio

We now carry out the sensitivity analysis of wholesale price and order quantities.

Proposition 1. $\frac{\partial w_3^*}{\partial \phi} > 0, \frac{\partial q_1^{3*}}{\partial \phi} < 0$ *and* $\frac{\partial q_2^{3*}}{\partial \phi} > 0$.

Proposition 1 shows that as the equity financing ratio increases, the wholesale price increases, retailer R_1's order quantity decreases but retailer R_2's order quantity increases. Note that a higher value of ϕ implies that the retailer borrows less from the bank and pays less interest to the bank. We can consider the equity financing ratio as a proxy for retailer R_2's operational costs. In this sense, as ϕ increases, retailer R_2 becomes more competitive in terms of costs and thus orders more from the supplier. Because of the strategic interaction with retailer R_2, retailer R_1 orders less.

Next, we compare the wholesale prices and order quantities across the three cases and highlight the role of interest rates and the equity financing ratio.

Proposition 2.

(i) If $r_f \le r_s$, *then* $w_2^* < w_1^* < w_3^*$; *if* $r_f > r_s$ *and* $\phi \le \phi_1 = \frac{r_f - r_s}{1+r_f}$, *then* $w_2^* < w_3^* \le w_1^*$; *if* $r_f > r_s$ *and* $\phi > \phi_1 = \frac{r_f - r_s}{1+r_f}$, *then* $w_2^* < w_1^* < w_3^*$;

(ii) If $q_1^{3*} > 0$ *and* $r_f \le r_s$, *then* $q_1^{3*} < q_1^{1*} < q_1^{2*}$ *and* $q_2^{1*} < q_2^{3*}$; *if* $q_1^{3*} > 0, r_f > r_s$ *and* $\phi < \phi_2 \cong \frac{r_f - r_s}{1+r_f}$, *then* $q_1^{1*} < q_1^{3*} < q_1^{2*}$ *and* $q_2^{3*} < q_2^{1*}$; *if* $q_1^{3*} > 0, r_f > r_s$ *and* $\phi > \phi_2 \cong \frac{r_f - r_s}{1+r_f}$, *then* $q_1^{3*} < q_1^{1*} < q_1^{2*}$ *and* $q_2^{1*} < q_2^{3*}$.

The supplier and retailer R_1 use the revenue sharing contract to coordinate the channel after their merger. Lemma 2 shows that such coordination requires a wholesale price below the production cost. Accordingly, the wholesale price w_2^* is the lowest among the three cases. In terms of the relationship between w_1^* and w_3^*, we observe three determinants: trade credit interest rate, risk-free interest rate, and the

equity financing ratio. If borrowing from the bank is cheaper, i.e., $r_f \leq r_s$, then the wholesale price is always higher in Case 1 than in Case 3. The intuition is as follows: because the bank interest rate is lower, retailer R_2 has cost advantages over retailer R_1. To leverage on retailer R_2's low cost, the supplier will increase the wholesale price. Thus, the wholesale price in Case 3 is higher than that in Case 1.

However, when the interest rate of trade credit is cheaper, i.e., $r_f > r_s$, whether retailer R_2 has competitive cost advantages depends on the equity financing ratio. We show that a threshold of the equity financing ratio ϕ_1 exists, such that if $\phi < \phi_1$, then it is in the supplier's interest to reduce the wholesale price to induce both retailers to buy more products. Hence, the wholesale price in Case 3 is lower than that in Case 1. By contrast, order quantities present a reversed pattern against wholesale prices, as shown in (*ii*).

Fig. 3 and 4 provide numerical examples that illustrate proposition 2. Fig. 3 describes how the supplier's wholesale price changes with the equity financing ratio ϕ. The main parameters are as follows: $A = 100$, $c = 10$, $\alpha = 0.18$ and $\gamma = 0.5$, where $r_f = 0.03$, $r_s = 0.05$ in Fig. 3(a) and $r_f = 0.08$, $r_s = 0.03$ in Fig. 3(b). Fig. 4 describes how the two retailers' order quantities change with ϕ. We observe that retailer R_1's order quantity is significantly higher in Case 2. When $r_f < r_s$, retailer R_1's order quantity in Case 3 is lower than that in Case 1,while retailer R_2's order quantity is higher than that in Case 1. We also observe that retailer R_1's order quantity decreases with ϕ and retailer R_2's order quantity increases with ϕ.When the equity financing ratio satisfies $\phi < 0.0463$, indicating that retailer R_2's main source of funds is bank loans, retailer R_2 has high repayment pressure in the case of $r_f > r_s$. Consequently, retailer R_2's order quantity is lower than that in Case 1, while retailer R_1's order quantity is higher than that in Case 1.

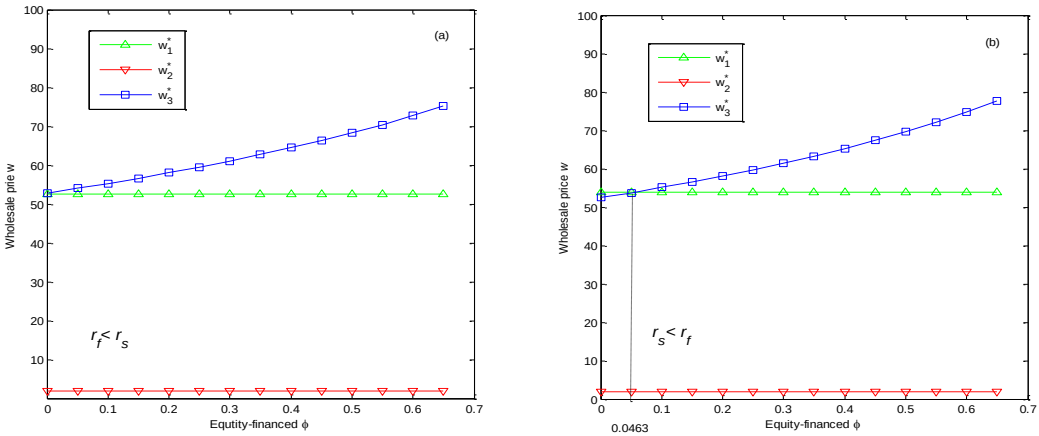


Fig. 3. The supplier's wholesale price changes with ϕ

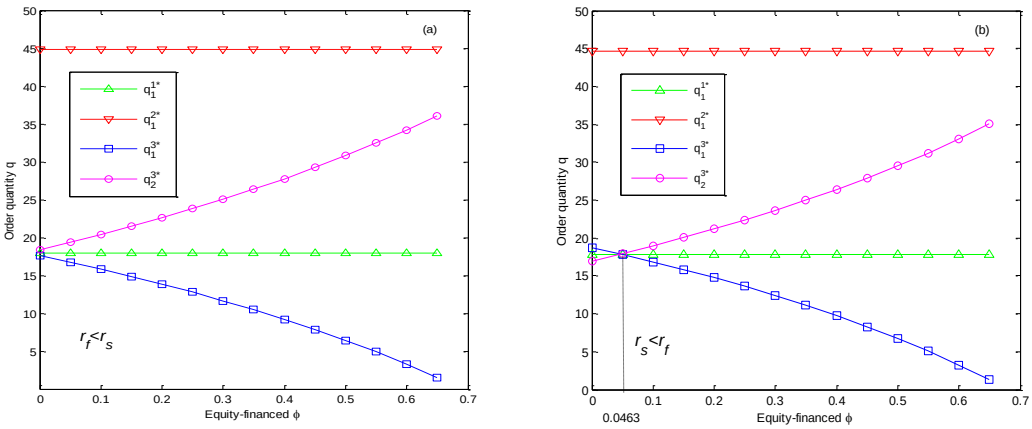


Fig. 4. The retailer's order quantity changes with ϕ

The above numerical examples illustrate how the equity financing ratio affects the equilibrium wholesale price and order quantities. We now examine the effect of the equity financing ratio on each player's profit. We also study, from the supplier's perspective, the condition under which retailer R_2 will be allowed to enter into the supply chain.

Proposition 3.

(i) $\frac{\partial\pi(R_1^3)}{\partial\phi} < 0$ and $\frac{\partial\pi(S^3)}{\partial\phi} > 0$; *a threshold* ϕ_3 *exists such that if* $\phi > \phi_3$, *then* $\pi(S^3) > \pi(S^2)$, *i.e., the supplier is willing to allow retailer* R_2 *to participate in the supply chain*;

(ii) An optimal equity finance ratio ϕ^* *exists for retailer* R_2.

Proposition 3 shows that retailer R_1's profit decreases with ϕ, while the supplier's profit increases with ϕ. The key condition under which the supplier is willing to

relinquish the merger and allow retailer R_2 to enter into the supply chain is the case in which the supplier makes a higher profit in Case 3 than in Case 2, i.e., $\pi(S^3) > \pi(S^2)$. We show that this condition is satisfied when the equity financing ratio is higher than a threshold ϕ_3. This implies that retailer R_2's cost advantage arising from a large portion of finance from investors will also benefit the supplier. Therefore, when ϕ is large, the supplier makes more profit in Case 3 than in Case 2.

The proof of Proposition 3 shows that retailer R_2's profit increases in the equity financing ratio ϕ for $\phi \in (0, \phi^*]$. As retailer R_2 transfers a large proportion of its sales revenue to investors when ϕ is large, retailer R_2's profit decreases with the equity financing ratio for $\phi \in (\phi^*, 1)$. Therefore, retailer R_2's profit first increases and then decreases with ϕ, and retailer R_2's profit is maximal at $\phi = \phi^*$.

Fig. 5 describes how the two retailers' profits change with ϕ. The parameters are as follows: $A = 100$, $c = 10$, $\gamma = 0.5$, $r_f = 0.03$, $r_s = 0.05$ and $\alpha = 0.18$. Fig. 5 shows that retailer R_1's profit decreases with ϕ in Case 3. Retailer R_1's profit is close to zero when ϕ is close to 0.7. Retailer R_2's profit increases with ϕ for $\phi \in (0, 0.55]$ and decreases with ϕ for $\phi \in (0.55, 1)$. The optimal equity financing ratio for retailer R_2 is $\phi^* = 0.55$.

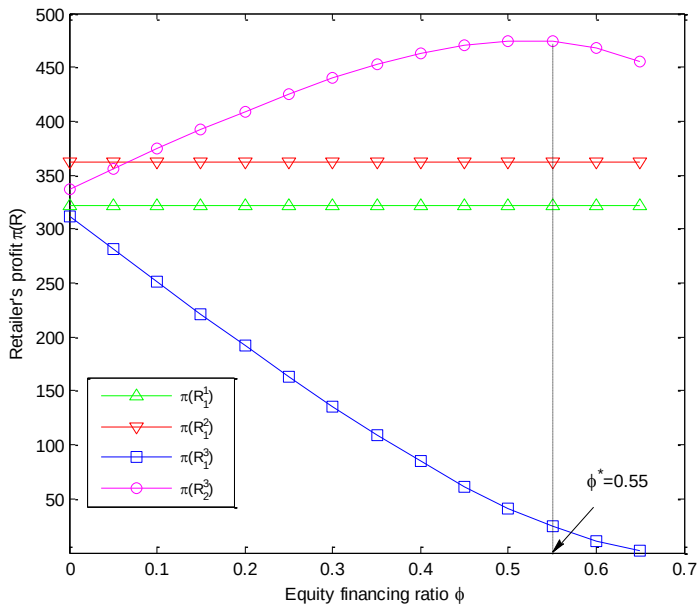


Fig. 5. The retailer's profit changes with ϕ

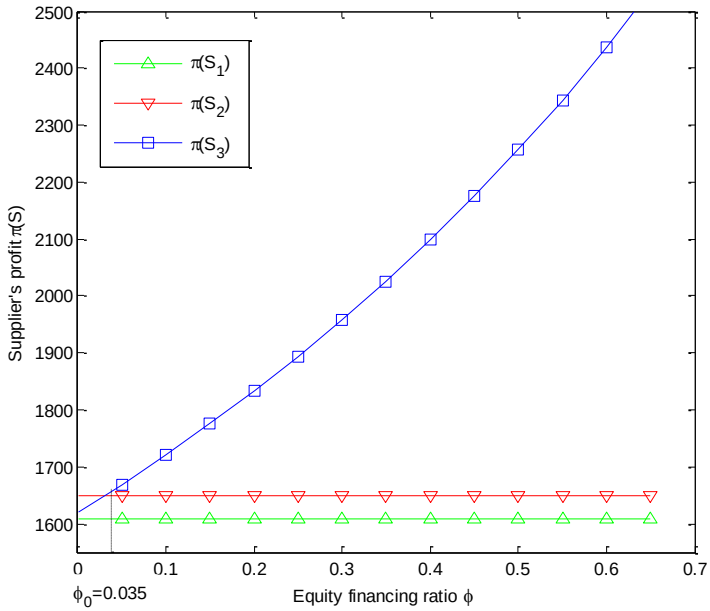


Fig. 6. The supplier's profit changes with ϕ

Fig. 6 illustrates how the supplier's profit changes with ϕ. It shows that the supplier's profit increases with ϕ in Case 3. Because the wholesale price and the total order quantity increase with ϕ, when the equity financing ratio is small and satisfies $\phi < 0.035$, the total order quantities are smaller. Therefore, the supplier's profit in Case 3 is less than that in Case 2. However, when the equity financing ratio ϕ is large and satisfies $\phi > 0.035$, the supplier's profit is greater in Case 3. Thus, the supplier will allow retailer R_2 to participate in the supply chain.

4.2 Competition Intensity

In this subsection, we examine the impact of competition intensity on the merger of the supplier and retailer R_1 (corresponding to Case 2). In doing so, we compare each player's profit and the total profit. In addition, this section examines how competition intensity affects the optimal wholesale price and the order quantities in each case.

Proposition 4 characterizes the impact of competition intensity on equilibrium evolution.

Proposition 4.

(i) If $0 \leq \gamma \leq \sqrt{2} - 1$, *then the supplier and retailer* R_1 *cannot merge.*

(ii) If $\sqrt{2}-1<\gamma\leq 1$ *and* $\frac{1}{(2+\gamma)^2}<\alpha<\frac{\gamma}{2+\gamma}$*, then the supplier and retailer* R_1 *are willing to merge with each other. If* $\sqrt{2}-1<\gamma\leq 1$ *and* $\alpha\leq\frac{1}{(2+\gamma)^2}$*, then retailer* R_1 *is not willing to merge with the supplier. If* $\sqrt{2}-1<\gamma\leq 1$ *and* $\alpha\geq\frac{\gamma}{2+\gamma}$*, then the supplier is not willing to merge with retailer* R_1*.*

We can show that in Case 1, as competition intensity increases, both the supplier's and the retailers' profits decrease. Proposition 4 shows that when γ is small, i.e., $0\leq\gamma\leq\sqrt{2}-1$, the supplier and retailer R_1 cannot merge. This is because the total profits after the merger are less than the sum of the supplier's and retailer R_1's profits before the merger. However, when γ is large, i.e., $\sqrt{2}-1<\gamma\leq 1$, the supplier and retailer R_1 may be willing to use a revenue sharing contract to achieve supply chain coordination. We find that whether the merger and coordination can be achieved depends on the revenue sharing rule between the supplier and retailer R_1. Specifically, as long as α is in the Pareto zone $(\frac{1}{(2+\gamma)^2},\frac{\gamma}{2+\gamma})$, the above revenue sharing contract renders a win-win result. Then, the supplier and retailer R_1 can merge; otherwise, the merger could not occur.

To illustrate the above findings, we now carry out numerical experiments. The main parameters are as follows: $A=100$, $c=10$, $\gamma=0.9$, $r_f=0.03$ and $r_s=0.05$. Fig. 7 depicts the Pareto zone under the revenue sharing contract in Case 2. As competition intensity increases, the Pareto zone expands, which implies that the likelihood of a successful merger between the supplier and retailer R_1 increases. Fig. 8 describes the changes in the supplier's profit as the competition intensity varies. The main parameters are as follows: $A=100$, $c=10$, $r_f=0.03$, $r_s=0.05$, $\alpha=0.18$ and $\phi=0.4$. As competition intensity increases, the supplier's profit decreases. When competitive intensity satisfies $0.439<\gamma\leq 1$, the supplier is willing to merge with retailer R_1.

Fig. 9 describes the changes in the total profits of the supplier and retailer R_1 with competition intensity. We use the same set of parameters employed in Fig. 8. We find that when competition intensity satisfies $0\leq\gamma\leq 0.358$, namely, the left side of

cyan dashed line, the supplier and retailer cannot merge. When competition intensity satisfies $0.358 < \gamma < 0.414$, namely, between the cyan dashed line and black dashed line, retailer R_1 is willing to merge but the supplier is not. Of note, when competition intensity satisfies $0.414 < \gamma \leq 1$, the total profits of the supplier and retailer R_1 in Case 2 are greater than those in Case 1. When competition intensity satisfies $0.414 < \gamma < 0.439$, namely, between the black dashed line and purple dashed line, retailer R_1 is willing to merge but the supplier is not. If and only if competition intensity satisfies $0.439 < \gamma \leq 1$, namely, the right side of the pink dashed line, both are willing to merge. In Case 3, the total profits of the supplier and retailer R_1 are always greater than those in Case 1.

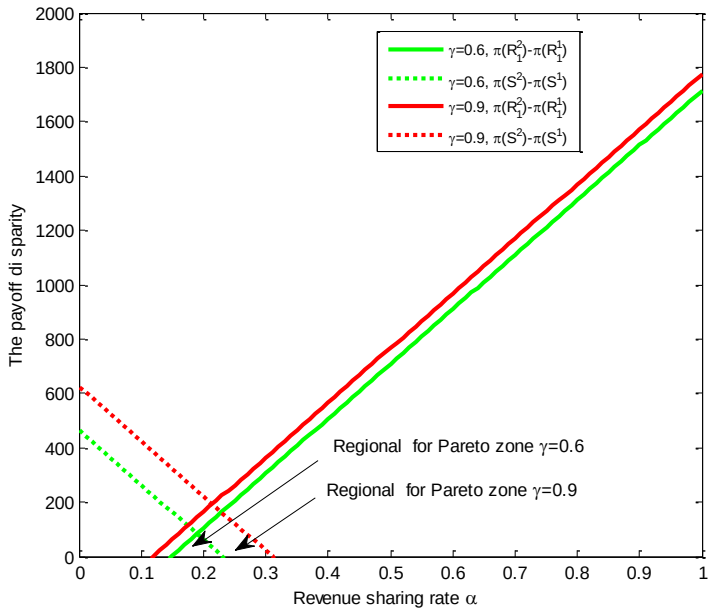


Fig. 7. The Pareto zone

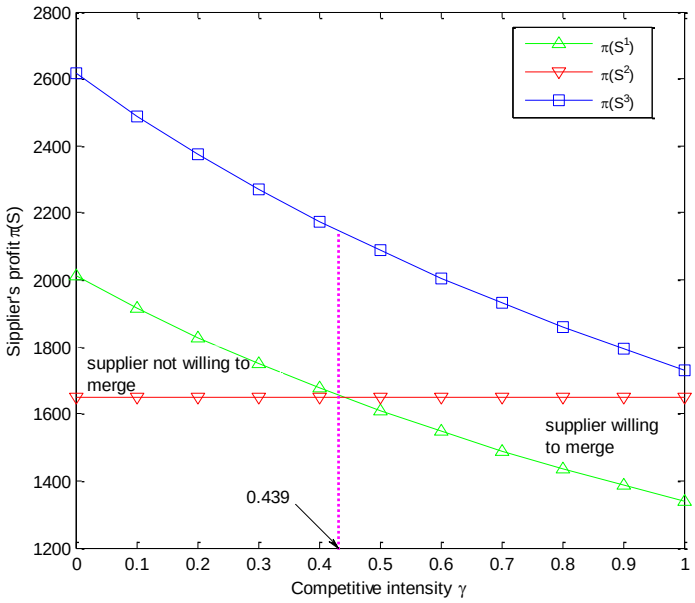


Fig. 8. The changes in the supplier's profit with *r*

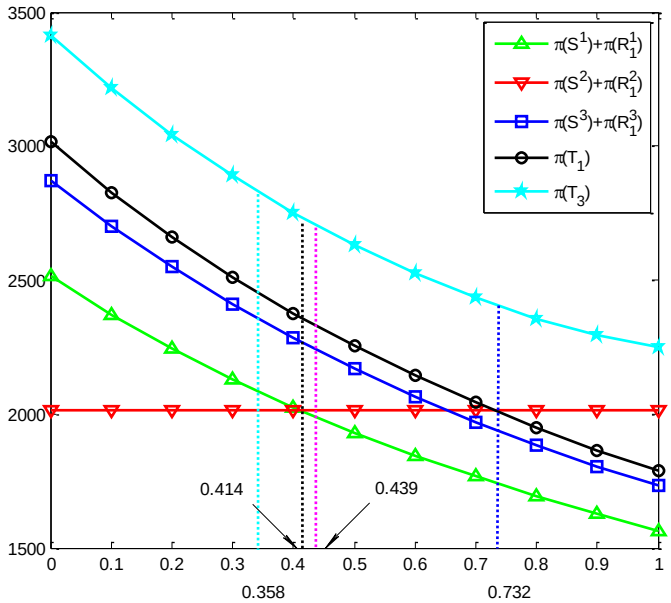


Fig. 9. The changes in the total profits of R_1 and the supplier with *r*

Proposition 5.

(i) $\frac{\partial w_1^*}{\partial \gamma} = 0$; $\frac{\partial w_2^*}{\partial \gamma} = 0$; *If* $r_f \le r_s$, *then* $\frac{\partial w_3^*}{\partial \gamma} \le 0$; *If* $r_f > r_s$ *and* $\phi \ge \phi_1 = \frac{r_f - r_s}{(1+r_f)}$, *then* $\frac{\partial w_3^*}{\partial \gamma} \ge 0$; *If* $r_f > r_s$ *and* $\phi < \phi_1 = \frac{r_f - r_s}{(1+r_f)}$, *then* $\frac{\partial w_3^*}{\partial \gamma} < 0$;

(ii) $\frac{\partial q_1^{1*}}{\partial \gamma} = \frac{\partial q_2^{1*}}{\partial \gamma} < 0$, $\frac{\partial q_1^{2*}}{\partial \gamma} = 0$.

As shown in Lemma 1, the two retailers have equal order quantities and sale prices in Case 1. This situation is equivalent to the case in which one retailer trades with the supplier. Therefore, the wholesale price is independent of competition intensity. When the supplier merges with retailer R_1, there is no competition in the market. Thus, the wholesale price is also independent of competitive intensity. In Case 3, if $r_f \leq r_s$, then retailer R_1's profit decreases with γ. The supplier will then reduce the wholesale price so that retailer R_1 does not to drop out of the supply chain. If $r_f > r_s$, we show that a threshold ϕ_1 exists such that if the equity financing ratio satisfies $\phi < \phi_1$, the wholesale price decreases with competition intensity. More interestingly, when the equity financing ratio is large, i.e., $\phi \geq \phi_1$, the supplier will increase the wholesale price as competition becomes more intensified. Proposition 5 also shows that in Case 1, as competition intensity increases, the two retailers' order quantities decrease.

Regarding Case 3, it is difficult to obtain analytical results for the sensitivity analysis in terms of order quantities and profits. Instead, we use numerical examples to examine how each retailer's order quantity and profit change with competition intensity. The main parameters are as follows: $A = 100$, $c = 10$, $r_f = 0.03$, $r_s = 0.05$ and $\alpha = 0.18$. Fig. 10 and 11 illustrate how retailer R_1's order quantity and profit change with competition intensity in the three cases. We observe that the order quantity and profit of retailer R_1 decrease with competition intensity in Case 3.

Fig. 12 and 13 describe how retailer R_2's order quantity and profit change with competition intensity and the equity financing ratio in Case 3. When the equity financing ratio is small, the order quantity and profit of retailer R_2 decrease with competitive intensity. Furthermore, when the equity financing ratio is small, retailer R_2's main source of funds is bank loans. As competition intensity increases, retailer R_2 will bear a high loan risk if it places large orders. As a result, retailer R_2 will reduce its order quantity. When the equity financing ratio is medium, the order quantity and profit of retailer R_2 first increase and then decrease with competition intensity. When the equity financing ratio is large, the order quantity and profit of retailer R_2 increase with competitive intensity.

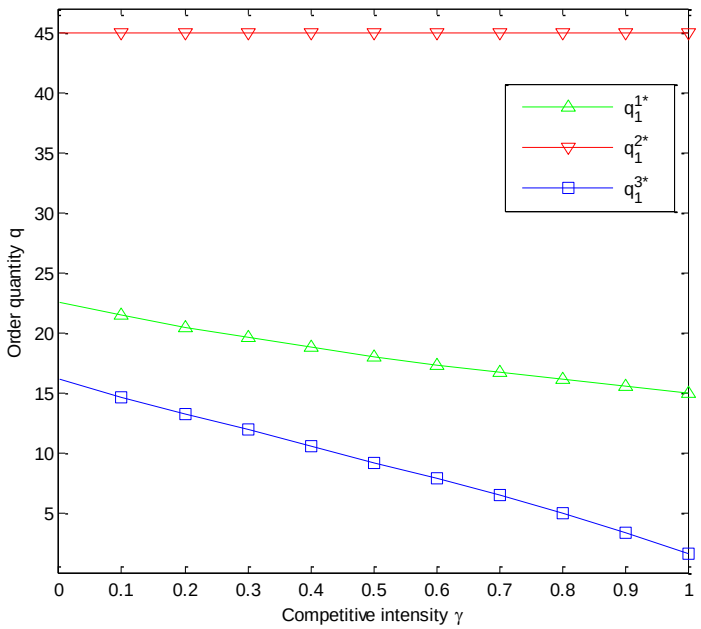


Fig. 10. The changes in R_1's order quantity with r

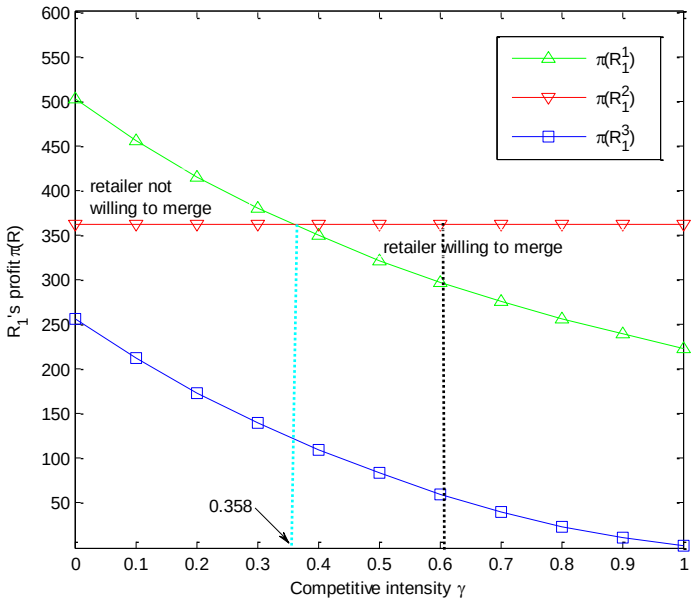


Fig. 11. The changes in R_1's profit with r

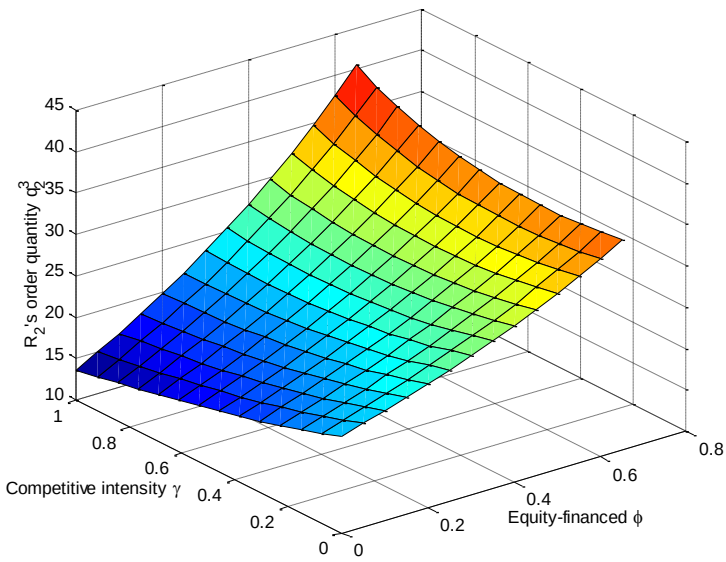


Fig. 12. The changes in R_2's order quantity in Case 3 with r and ϕ

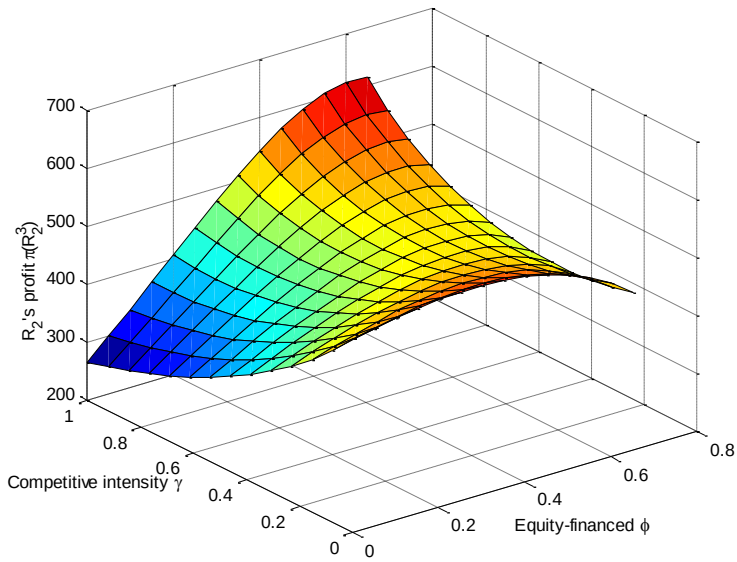


Fig. 13. The changes in R_2's profit in Case 3 with r and ϕ

5. Conclusion

In this paper, we consider a game theoretical model for a supply chain with a supplier and two capital constrained retailers under Cournot competition. The supplier may extend trade credit to these retailers. We consider three cases based on how many retailers receive trade credit from the supplier and whether retailers have access to external financing (including equity and bank financing): in Case 1, the supplier

provides trade credit to both retailers; in Case 2, the supplier provides trade credit to only one retailer and the other retailer has no other financing options. In this case, the supplier and one of the retailers sign a revenue sharing contract and these two players achieve supply chain coordination; and in Case 3, the supplier provides trade credit to one retailer but the other retailer is able to use external financing, including equity financing and debt financing. Our research differs from the existing literature in that we consider a competitive setting in which financially constrained retailers compete in a consumer market. We also explicitly incorporate equity financing into our model, enabling us to study how equity financing affects the equilibrium decisions of the supplier and the retailers.

With the above setup, we find some useful results that complement the existing literature. First, we show that when competition intensity is below a threshold, merger cannot be achieved because the total supply chain profit is simply smaller after the merger. Therefore, the supplier and the retailer cannot benefit from the merger. However, when competition intensity is above the threshold, the supply chain profit is greater after the merger, and whether both the supplier and the retailer are willing to merge depends on the revenue sharing ratio. We find a Pareto zone of the ratio in which a merger can be achieved. Second, we find that the retailer who drops out of the supply chain can break the merger of the supplier and the rival by using equity financing and debt financing. Specifically, when the equity financing ratio is higher than a threshold, the supplier is willing to relinquish the merger and allow the second retailer to enter into the supply chain. This is because retailer R_2's cost advantage arising from a larger portion of finance from investors will also benefit the supplier. Thus, when the ratio is sufficiently large, the merger will be broken by the entrance of the second retailer. There exists an optimal equity finance ratio for the second retailer. We also analyze how different parameters affect the order quantities, wholesale prices and players' profits in each scenario.

Before we conclude, we present some potential extensions. For example, the current paper considers a deterministic demand model. However, the demand for some new products may be highly uncertain, and this is directly tied with the

investors' and the banks' perceptions of the retailers' financial risks. Therefore, further studies should investigate how demand uncertainty shapes the existing results.

Acknowledgments

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Appendix

Proof of Lemma 1.

Substituting $p_i^1 = A - q_i^1 - \gamma q_j^1$ into equation (1) yields $\pi(R_i^1) = -{q_i^1}^2 + (A - \gamma q_j^1 - w_1(1 + r_s))q_i^1$. It is straightforward to show that $\pi(R_i^1)$ is a convex function with respect to q_i^1. Solving the first-order condition, we get the order quantity$q_i^1 = \frac{1}{(2+\gamma)}\left(A - w_1(1 + r_s)\right)$, where $i = 1,2$. Substituting $q_i^1 = \frac{1}{(2+\gamma)}\left(A - w_1(1 + r_s)\right)$ into equation (2), we obtain $\pi(S^1) = \frac{1}{(2+\gamma)}[{w_1}^2(1 + r_s)^2 + \left(A + c\left(1 + r_f\right)\right)w_1(1 + r_s) - Ac\left(1 + r_f\right)]$. We easily show that$\pi(S^1)$is a convex function with respect tow_1. Solving the first-order condition, we get the optimal wholesale price $w_1^* = \frac{1}{2(1+r_s)}\left(A + c(1 + r_f)\right)$. Substituting it into $q_i^1 = \frac{1}{(2+\gamma)}\left(A - w_1(1 + r_s)\right)$, we obtain $q_i^{1*} = \frac{1}{2(2+\gamma)}\left(A - c(1 + r_f)\right)$. □

Proofs of Lemma 2 and Lemma 3.

Proofs of Lemma 2 and Lemma 3 are similar to that of Lemma 1 and, hence, are

omitted. □

Proof of Proposition 1.

The first-order condition of w_3^* with respect to ϕ is given by $\frac{\partial w_3^*}{\partial \phi} = \frac{(1+r_f)(2-\gamma)}{2[B(1+r_s)+D(1+r_f)]^2}\{EA-(2+\gamma)(1+r_s)(r_s-r_f)c\}$, where $E = (2+r_f+r_s)[2(1+r_f)-\gamma(1+r_s)]$. $\frac{\partial w_3^*}{\partial \phi} > \frac{(1+r_f)(2-\gamma)(1+r_s)}{2[B(1+r_s)+D(1+r_f)]^2}\{[2(1+r_f)-(1+r_s)]A-(2+1)(r_s-r_f)c\} = \frac{(1+r_f)(2-\gamma)(1+r_s)}{2[B(1+r_s)+D(1+r_f)]^2}\{[1+2r_f-r_s]A-3(r_s-r_f)c\}$. We easily show $[1+2r_f-r_s]A-3(r_s-r_f)c > 0$. Thus, we have $\frac{\partial w_3^*}{\partial \phi} > 0$.

Because $\frac{\partial q_1^{3*}}{\partial \phi} = \frac{\left(-\gamma(1+r_f)w_3^*+B\frac{\partial w_3^*}{\partial \phi}\right)}{4-\gamma^2}$, $B < 0$ and $\frac{\partial w_3^*}{\partial \phi} > 0$, we have $\frac{\partial q_1^{3*}}{\partial \phi} < 0$. $\frac{\partial q_2^{3*}}{\partial \phi} = \frac{(1+r_f)[FA+Hc(1+r_f)]}{2(2+\gamma)[B(1+r_s)+D(1+r_f)]^2}$ where $F = (4-\gamma^2)(1+r_s)^2(2+r_f+r_s)$ and $H = \{(-\gamma^2-2\gamma+4)(1+r_s)^3+(\gamma^2+8(1-\phi))(1+r_f)(1+r_s)^2-2\gamma(1-\phi)(3-\phi)(1+r_f)^2(1+r_s)+4(1-\phi)^2(1+r_f)^3\}$. We derive that $H > 0$ and $F > 0$. Accordingly, we have $\frac{\partial q_2^{3*}}{\partial \phi} > 0$. □

Proof of Proposition 2.

(i) Base on the results in Lemma 1 and 3, we have $w_3^* - w_1^* = \frac{(r_s-r_f)Dc(1+r_f)-[(1+r_s)-(1-\phi)(1+r_f)][2(1+r_f)-\gamma(1+r_s)]A}{2(1+r_s)(B(1+r_s)+D(1+r_f))}$. Because $D = \gamma(1+r_s)-2(1-\phi)(1+r_f) < (1+\mathrm{r_s})-(1-\phi)(1+r_f)$, we have

$w_3^* - w_1^* > \frac{[(1+r_s)-(1-\phi)(1+r_f)][(r_s-r_f)c(1+r_f)-[2(1+r_f)-\gamma(1+r_s)]A]}{2(1+r_s)(B(1+r_s)+D(1+r_f))}$. Because $B(1+r_s)+D(1+r_f) = \gamma(2-\phi)(1+r_f)(1+r_s)-2(1+r_s)^2-2(1-\phi)(1+r_f)^2 < 0$, we have that if $r_f \le r_s$, then $w_3^* > w_1^*$.

Let $(1+\mathrm{r_s})-(1-\phi)(1+r_f) = 0$.We obtain $\phi = \phi_1 = \frac{r_f-r_s}{1+r_f}$. If $r_f > r_s$ and $\phi > \phi_1$, then $\frac{[(1+r_s)-(1-\phi)(1+r_f)][(r_s-r_f)c(1+r_f)-[2(1+r_f)-\gamma(1+r_s)]A]}{2(1+r_s)(B(1+r_s)+D(1+r_f))} > 0$. Accordingly,$w_3^* > w_1^*$. If $r_f > r_s$ and $\phi \le \phi_1$, then $w_3^* - w_1^* = \frac{(r_s-r_f)[\gamma(1+r_s)-2(1-\phi)(1+r_f)]c(1+r_f)-[(1+r_s)-(1-\phi)(1+r_f)][2(1+r_f)-\gamma(1+r_s)]A}{2(1+r_s)(B(1+r_s)+D(1+r_f))}$.

Thus, we have $w_3^* < w_1^*$.

Proof of Proposition 2 (ii) is similar to that of Proposition 2 (i) and, hence, is omitted. □

Proof of Proposition 3.

(i) Proofs of $\frac{\partial\pi(R_1^3)}{\partial\phi} < 0$ and $\frac{\partial\pi(S^3)}{\partial\phi} > 0$ are similar to that of Proposition 1.

Let $\pi(S^3) - \pi(S^2) > 0$. We have $\phi > \phi_3$ where ϕ_3 satisfies the equation $[(2-\gamma)^2(2+r_f+r_s)^2 - (1-\alpha)(4-\gamma^2)\left(B(1+r_s)+D\left(1+r_f\right)\right)]A^2 + 2(2-\gamma)[B((1+r_f) + \left(5-(2+\gamma)(1-\alpha)(1+r_s)\right)) + D\left((1+r_s)+(5-(2+\gamma)(1-\alpha)(1+r_f)\right)]Ac(1+r_f) + [(B+D)^2 - (2+\gamma)(1-\alpha)\left(B(1+r_s)+D\left(1+r_f\right)\right)]\left(c\left(1+r_f\right)\right)^2 = 0$.

(ii) Let $\frac{\partial\pi(R_2^3)}{\partial\phi} = -q_2^{3*}g(\phi)$, where $g(\phi) = q_2^{3*} - 2(1-\phi)\frac{\partial q_2^{3*}}{\partial\phi}$. We have $\frac{\partial g(\phi)}{\partial\phi} = 3\frac{\partial q_2^{3*}}{\partial\phi} - 2(1-\phi)\frac{\partial^2 q_2^{3*}}{\partial\phi^2}$. As $\frac{\partial^2 q_2^{3*}}{\partial\phi^2} = \frac{(2-\gamma)\left(1+r_f\right)^2(1+r_s)^2\ [MA+Nc\left(1+r_f\right)]}{-\left[B(1+r_s)+D\left(1+r_f\right)\right]^3}$, where $M = 2\left(1+r_f\right)^2 + (2-\gamma)\left(1+r_f\right)(1+r_s) - \gamma(1+r_s)^2$ and $N = (2+\gamma)(1+r_s)(r_f - r_s)$, we have $\frac{\partial g(\phi)}{\partial\phi} = \frac{\left(1+r_f\right)\{(3\left[B(1+r_s)+D\left(1+r_f\right)\right]F+4(4-\gamma^2)(1-\phi)\left(1+r_f\right)(1+r_s)^2M)A + (3\left[B(1+r_s)+D\left(1+r_f\right)\right]H-4(4-\gamma^2)(1-\phi)\left(1+r_f\right)(1+r_s)^2N)c\left(1+r_f\right)\}}{2(2+\gamma)\left[B(1+r_s)+D\left(1+r_f\right)\right]^3}$, where $3\left[B(1+r_s)+D\left(1+r_f\right)\right]F + 4(4-\gamma^2)(1-\phi)\left(1+r_f\right)(1+r_s)^2M = (4-\gamma)^2(1+r_f)(1+r_s)^2[-6(1+r_s)^3 + 2(1-\phi)\left(1+r_f\right)^3 + [3\gamma+(2-\gamma)(1-\phi)]\left(1+r_f\right)^2(1+r_s) - [(2-\gamma)+\gamma(1-\phi)]\left(1+r_f\right)(1+r_s)^2]$.

Let $h(\gamma) = -6(1+r_s)^3 + 2(1-\phi)\left(1+r_f\right)^3 + [3\gamma+(2-\gamma)(1-\phi)]\left(1+r_f\right)^2(1+r_s) - [(2-\gamma)+\gamma(1-\phi)]\left(1+r_f\right)(1+r_s)^2$. We have $\frac{\partial h(\gamma)}{\partial\gamma} > 0$. Because $h(1) = -6(1+r_s)^3 + 2(1-\phi)\left(1+r_f\right)^3 + (4-\phi)\left(1+r_f\right)^2(1+r_s) - (2-\phi)\left(1+r_f\right)(1+r_s)^2 < 0$ and $\frac{\partial h(\gamma)}{\partial\gamma} > 0$, then $h(\gamma) < 0$. Similarly, we have $3\left[B(1+r_s)+D\left(1+r_f\right)\right]H - 4(4-\gamma^2)(1-\phi)\left(1+r_f\right)(1+r_s)^2N)c\left(1+r_f\right) < 0$. Hence, we derive $\frac{\partial g(\phi)}{\partial\phi} > 0$ and $g(0) = \frac{XA+Yc\left(1+r_f\right)}{2(2+\gamma)\left[B(1+r_s)+D\left(1+r_f\right)\right]^2}$, where $X = [B(1+r_s) + D(1+r_f)](B-D)(1+r_s) - 2\left(1+r_f\right)(1+r_s)^2F = 2(2+\gamma)(1+r_s)[(1+r_s)^3 -$

$(1+r_f)^3-(1-2\gamma)\left(1+r_f\right)^2(1+r_s)]-3\left(1+r_f\right)(1+r_s)^2<0$ and $Y=-D[D(1+r_f)+B(1+r_s)]\left(2+r_f+r_s\right)-2\left(1+r_f\right)H$. We find that $Y<0$ and $g(0)<0$. As $g(1)=q_2^{3*}>0$ and $g(\phi)$ increases in ϕ, there exists a ϕ^* over (0,1) that satisfies the equation $g(\phi^*)=0$. Solving $g(\phi^*)=0$ gets ϕ^*, satisfying the equation $\left[B(1+r_s)+D\left(1+r_f\right)\right]\left[D\left(r_f-r_s\right)+2B(1+r_s)-2\left(1+r_f\right)(1-\phi)F\right]A=\{[B(1+r_s)+D\left(1+r_f\right)]\left[(1-\phi)\left(1+r_f\right)+(1+r_s)\right]D+2\left(1+r_f\right)(1-\phi)H\}c\left(1+r_f\right)$. Hence, we obtain $g(\phi)<0$ over $(0,\phi^*]$ and $g(\phi)>0$ over $(\phi^*,1)$.

As $\frac{\partial\pi\left(R_2^3\right)}{\partial\phi}=-q_2^{3*}g(\phi)$, we have $\frac{\partial\pi\left(R_2^3\right)}{\partial\phi}>0$ over $(0,\phi^*]$ and $\frac{\partial\pi\left(R_2^3\right)}{\partial\phi}<0$ over $(\phi^*,1)$. Therefore, retailer R_2's profit is at the maximal point at $\phi=\phi^*$, which means ϕ^* is retailer R_2's optimal equity financing ratio. □

Proof of Proposition 4.

(i) The total profits of the supplier and retailer R_1 is $\pi(R_1^1)+\pi(S^1)=\frac{5+2\gamma}{4(2+\gamma)^2}\left(A-c\left(1+r_f\right)\right)^2$ in Case 1. The total profits of the supplier and retailer R_1 is $\pi(R_1^2)+\pi(S^2)=\frac{1}{4}\left(A-c\left(1+r_f\right)\right)^2$ in Case 2. The use of a simple algebraic transformation gives $(\pi(R_1^2)+\pi(S^2))-(\pi(R_1^1)+\pi(S^1))=\frac{\gamma^2+2\gamma-1}{4(2+\gamma)^2}\left(A-c\left(1+r_f\right)\right)^2$. If $\gamma^2+2\gamma-1>0$, then we have $\pi(R_1^2)+\pi(S^2)>\pi(R_1^1)+\pi(S^1)$. Thus, we obtain $\sqrt{2}-1<\gamma\leq 1$. The total profits of the supplier and retailer R_1 after merger is greater than that in Case 1 when $\sqrt{2}-1<\gamma\leq 1$. So when $0\leq\gamma\leq\sqrt{2}-1$, the supplier and retailer R_1 cannot merge.

The total profit of the supply chain in Case 1 is $\pi(T_1)=\frac{3+\gamma}{2(2+\gamma)^2}\left(A-c(1+r_f)\right)^2$. The total profit of the supply chain in Case 2 is $\pi(T_2)=\frac{1}{4}\left(A-c\left(1+r_f\right)\right)^2$. Using a simple algebraic transformation, we obtain $\pi(T_2)-\pi(T_1)=\frac{\gamma^2+2\gamma-2}{4(2+\gamma)^2}\left(A-c\left(1+r_f\right)\right)^2$. If $\gamma^2+2\gamma-2>0$, i.e., $\sqrt{3}-1<\gamma\leq 1$, then $\pi(T_2)>\pi(T_1)$. Thus, we show that the total profit of the supply chain after merger is greater than the total

profit of the supply chain in Case 1 when $\sqrt{3}-1<\gamma\leq 1$.

(ii) We have $\pi(R_1^2)-\pi(R_1^1)=\frac{1}{4}(\alpha-\frac{1}{(2+\gamma)^2})\left(A-c\left(1+r_f\right)\right)^2$ and $\pi(S^2)-\pi(S^1)=\frac{1}{2}(\frac{1-\alpha}{2}-\frac{1}{2+\gamma})\left(A-c\left(1+r_f\right)\right)^2$. If $\frac{1}{(2+\gamma)^2}<\alpha<\frac{\gamma}{2+\gamma}$, then $\pi(R_1^2)-\pi(R_1^1)>0$ and $\pi(S^2)-\pi(S^1)>0$. In this case, the supplier and retailer R_1 are willing to merge with each other because they can obtain more profit after the merger. If $\alpha<\frac{1}{(2+\gamma)^2}$, then $\pi(R_1^2)-\pi(R_1^1)<0$ and $\pi(S^2)-\pi(S^1)>0$. In this case, retailer R_1's profit is reduced, while the supplier's profit is increased after merger. If $\alpha>\frac{\gamma}{2+\gamma}$, then $\pi(R_1^2)-\pi(R_1^1)>0$ and $\pi(S^2)-\pi(S^1)<0$. Thus, the supplier's profit is reduced after the merger, while retailer R_1's profit is increased. □

Proof of Proposition 5.

(i) Lemma 1 and 2 show that w_1^* and w_2^* is independent of γ. Hence, we can obtain $\frac{\partial w_1^*}{\partial\gamma}=0$ and $\frac{\partial w_2^*}{\partial\gamma}=0$. The first derivative of w_3^* with respect to γ yields $\frac{\partial w_3^*}{\partial\gamma}=\frac{K[(2+r_f+r_s)A+[(1-\phi)(1+r_f)+(1+r_s)]c(1+r_f)]}{2[B(1+r_s)+D(1+r_f)]^2}$, where $K=\left(B(1+r_s)+D\left(1+r_f\right)\right)+(2-\gamma)(2-\phi)\left(1+r_f\right)(1+r_s)$ and $B(1+r_s)+D\left(1+r_f\right)=(2-\phi)\,\gamma\left(1+r_f\right)(1+r_s)-2(1+r_s)^2-2(1-\phi)\left(1+r_f\right)^2$. Accordingly, $K=2(2-\phi)\left(1+r_f\right)(1+r_s)-2(1+r_s)^2-2(1-\phi)\left(1+r_f\right)^2=2\left[(1-\phi)\left(1+r_f\right)-(1+r_s)\right]\left(r_s-r_f\right)$. Thus, if $r_f<r_s$, then $\frac{\partial w_3^*}{\partial\gamma}<0$. If $r_f=r_s$, then $\frac{\partial w_3^*}{\partial\gamma}=0$. If $r_f<r_s$ and $0<\phi\leq\frac{r_f-r_s}{(1+r_f)}$, then $\frac{\partial w_3^*}{\partial\gamma}\leq 0$. If $r_f<r_s$ and $\phi>\frac{r_f-r_s}{(1+r_f)}$, then $\frac{\partial w_3^*}{\partial\gamma}>0$.

(ii) $\frac{\partial q_1^{1*}}{\partial\gamma}=\frac{\partial q_2^{1*}}{\partial\gamma}=-\frac{1}{2(2+\gamma)^2}\left(A-c(1+r_f)\right)<0$. □

References

Arya, A., Mittendorf, B., Sappington, D.E.M., 2008. Outsourcing, vertical integration, and price vs. quantity competition. Int. J. Ind. Organ. 26 (1), 1-16.

Arya, A., Mittendorf, B., 2013. The changing face of distribution channels: partial forward integration and strategic investments. Prod. Oper. Manag. 22 (5),

1077-1088.

Brander, J.A., Lewis, T.R., 1986. Oligopoly and financial structure: the limited liability effect. Am. Econ. Rev. 76 (5), 965-970.

Brander, J.A., Lewis, T.R., 1988. Bankruptcy costs and the theory of oligopoly. Can. J. Econ. 21 (2), 221-243.

Cachon, G.P., 2003. Supply chain coordination with contracts. Handbooks in Operations Research and Management Science: Supply Chain Management. North-Holland, pp. 1-126.

Cachon, G.P., Lariviere, M.A., 2005. Supply chain coordination with revenue-sharing contracts: strength and limitations. Manag. Sci. 51 (1), 30-44.

Cho, S.H., 2014. Horizontal mergers in multitier decentralized supply chains. Manag. Sci. 60 (2), 356-379.

Chen, X.F., 2015. A Model of Trade Credit in a Capital-constrained Distribution Channel. Int. J. Prod. Econ. 159, 347-357.

Du, R., Banerjee, A., Kim, S.L., 2013. Coordination of two-echelon supply chains using wholesale price discount and credit option. Int. J. Prod. Econ. 143 (2), 327-334.

Fang, Y.E., Shou, B.Y., 2015. Managing supply uncertainty under supply chain Cournot competition. Eur. J. Oper. Res. 243 (1), 156-176.

Feng, X.H., Moon, I., Ryu, K., 2015. Supply chain coordination under budget constraints. Comput. Ind. Eng. 88, 487-500.

Glock, C.H., Kim, T., 2015. The effect of forward integration on a single-vendor–multi-retailer supply chain under retailer competition. Int. J. Prod. Econ. 164, 179-192.

Goyal, S.K., 1985. Economic order quantity under conditions of permissible delay in payments. J. Oper. Res. Soc. 36 (4), 335-338.

Huang,Y.F., 2007. Economic order quantity under conditionally permissible delay in payments. Eur. J. Oper. Res.176 (2), 911-924.

Jing, B., Chen, X.F., Cai, G.S., 2012. Equilibrium financing in a distribution channel with capital constraint. Prod. Oper. Manag. 21 (6), 1090-1101.

Jing, B., Seidmann, A., 2014. Finance sourcing in a supply chain. Decis Support Syst. 58 (1), 15-20.

Jin, Y.N., Wang, S.J., Hu, Q.Y., 2015. Contract type and decision right of sales promotion in supply chain management with a capital constrained retailer. Eur. J. Oper. Res. 240 (2), 415-424.

Kouvelis, P., Zhao, W.H., 2012. Financing the Newsvendor: Supplier vs. Bank, and the Structure of Optimal Trade Credit Contracts. Oper. Res. 60 (3), 566-580.

Lee, C.H., Rhee, B.D., 2011. Trade credit for supply chain coordination. Eur. J. Oper. Res. 214 (1), 136-146.

Lee, C.H., Rhee, B.D., 2010. Coordination contracts in the presence of positive inventory financing costs. Int. J. Prod. Econ. 124 (2), 331-339.

Li, G., Huang, F.F., Cheng, T.C.E., Ji, P., 2015. Competition between manufacturer's online customization channel and conventional retailer. IEEE Trans. Eng. Manage. 62 (2), 150-157.

Ru, J., Wang, Y., 2010. Consignment contracting: who should control inventory in the supply chain? Eur. J. Oper. Res. 201 (3), 760-769.

Seifert, D., Seifert, R.W., Protopappa-Sieke, M., 2013. A review of trade credit literature: Opportunities for research in operations? Eur. J. Oper. Res. 231 (2), 245-256.

Spengler, J.J., 1950. Vertical integration and antitrust policy. J. Polit. Econ. 58 (4), 347-352.

Teng, J.T., Min, J., Pan, Q.H., 2012. Economic order quantity model with trade credit financing for non-decreasing demand. Omega. 40 (3), 328-335.

Taleizadeh, A.A., Pentico, D.W., Jabalameli, M.S., Aryanezhad, M., 2013. An EOQ model with partial delayed payment and partial backordering. Omega. 41 (2), 354-368.

Wang, W., Li, G., Cheng, T.C.E., 2016. Channel selection in a supply chain with a multi-channel retailer: the role of channel operating costs. Int. J. Prod. Econ. 173, 54-65.

Wu, C.H., Chen, C.W., Hsieh, C.C., 2012. Competitive pricing decisions in a two-echelon supply chain with horizontal and vertical competition. Int. J. Prod. Econ. 135 (1), 265-274.

Xu, X.Y., Cheng, X.Y., Sun, Y.H., 2015. Coordination contracts for outsourcing supply chain with financial constraints. Int. J. Prod. Econ. 162, 134-142.

Yang, S., Hong, K.S., Lee, C., 2014. Supply chain coordination with stock-dependent demand rate and credit incentives. Int. J. Prod. Econ. 157, 105-111.

Yan, N.N., Sun, B.W., 2013. Coordinating loan strategies for supply chain financing with limited credit. OR Spectrum. 35 (4), 1039-1058.

Yang, S.L., Zhou, Y.W., 2006. Two-echelon supply chain models: Considering duopolistic retailers different competitive behaviors. Int. J. Prod. Econ. 103 (1), 104-116.

Zhang, Q.H., Dong, M., Luo, J.W., Segerstedt, A., 2014. Supply chain coordination with trade credit and quantity discount incorporating default risk. Int. J. Prod. Econ. 153, 352-360.

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